A GENERALIZED SOLUTION FOR SLIDING MODE CONTROL

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ABSTRACT: In the paper, a general method for applying the sliding mode control to the electronic power converters operating as variable structure nonlinear systems is developed. Unlike the method proposed by Bühler (Bühler 1986), this one may be aplied even if the reactive elements (L, C) of the converter are placed in front as well as behind the switch/switches. A new block diagram and a switching function for the variable structure systems are proposed. Everyone of the state equations systems matrices is written as a sum of two matrices and a matrix in every sum is multiplied by the switching function. The state equations of the overall "convertor + load " circuit, the switching law, the equivalent control signal, the existence conditions and the switching frequency are presented. The method is applied to three particular sliding mode control power converters.

KEYWORDS: *sliding mode control, power electronic converters, switching function*

1. INTRODUCTION

Generally, the power electronic converters are circuits embodying diodes and static switches, consisting in conventional thyristors, gate turn-off thyristors, bipolar junction transistors, power MOS-FETs, insulated gate bipolar transistors (IGBT), passive elements (R, L, C) and independent voltage and current generators. In these circuits, after every commutation of the switches, the equivalent structure of the circuit changes. In the same time, different nonlinearities appear (relay type nonlinearities, hysteresis type nonlinearities, etc). These circuits are called "variable structure nonlinear systems".

A very suitable method for the control of this type of systems is the sliding mode control.

2. PROPOSED SOLUTION

In (Bühler 1986) for a variable structure system using sliding mode control, a block diagram model is proposed, where the power converter is represented by a switch and its reactive elements are included in the load. This diagram is useful only if the reactive elements (L, C) of the converter are placed behind the switch (for instance, the dc-dc Buck converter). The power converters possessing reactive elements in front of and behind the switch/switches cannot be described by this diagram. Such converters are the dc-dc Boost converter and the single phase inverter active filter. For analyzing the systems of this type, we propose the general block diagram in fig.1.

In the block diagram in fig.1, the power block is no more a switch but a power converter possessing static switches and passive elements (R, L, C). In a power converter there are two variables determining its state: the supply voltage "e" and the control signal of the converter static switches. Therefore, we consider the power converter to be characterized by a switching function "z" which takes into account the state of the circuit switches; this state is determined by the comparator, that is by the actuating signal.

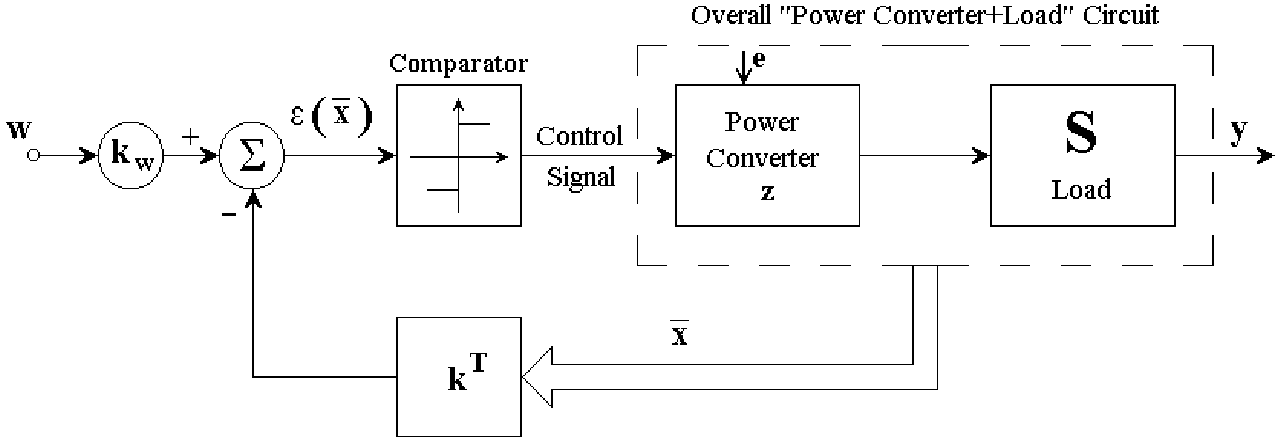

Fig.1. The block diagram of a variable structure nonlinear system using sliding mode control

The switching function allows the writing of the state equations not only for the load S, but for the overall circuit "power converter + load".

The state vector for the overall "power converter + load" circuit is composed of the state variables of the load S (the vector $\overline{x_s}$) and the state variables introduced by the reactive elements (L, C) of the converter (the vector $\overline{x_c}$).If we denote by $\overline{x}$ the state vector of the overall "power converter + load" circuit, we can write its state equations system:

$$\begin{cases} \dot{\overline{x}} = A\overline{x} + be \\ y = c^T \overline{x} \end{cases} \qquad (1)$$

where:

$$\overline{x} = \left| \overline{x_s} \quad \overline{x_c} \right|^T ;$$

e - the input (generally, the supply voltage of the converter);

A - a quadratic matrix depending of the elements (R, L, C) of the overall "converter + load" circuit and the switching function "z".The quadratic matrix A can be splited in two quadratic matrices A_1 and A_2 which depend on the elements (R, L, C) of the overall "converter + load" circuit, the matrix A_2 being multiplied by the switching function "z" in the following way:

$$A = A_1 + zA_2 \qquad (2)$$

b - a column vector determined by the elements (R, L, C) of the overall "converter + load" circuit and the switching function "z". The column vector "b" can be splited in two column vectors b_1 and b_2 which are determined by the passive elements of the overall "converter + load" circuit, the vector b_2 being multiplied by the switching function "z":

$$b = b_1 + zb_2 \qquad (3)$$

$c^T = \left(c_s^T \quad c_c^T \right)$, where c_s^T corresponds to the vector $\overline{x_s}$ and c_c^T corresponds to the vector $\overline{x_c}$.

- **The switching law**

From fig.1. results:

$$\varepsilon\left(\overline{x}\right) = -k^T \overline{x} + k_w w \qquad (4)$$

- **The equivalent control signal**

Substituting the vector $\dot{\overline{x}}$, as it is expressed in (1), in the derivative of the switching law (4) and equating it to zero, results:

$$-k^T \left(A\overline{x} + be\right) + k_w \dot{w} = 0$$

Substituting in this equation A and b as they are expressed in (2) and (3) respectively and taking into account that $z = z_{equiv}$, the equivalent control signal is obtained:

$$z_{equiv} = \frac{1}{k^T (A_2 \overline{x} + b_2 e)} \cdot \left[-k^T (A_1 \overline{x} + b_1 e) + k_w \dot{w} \right] \qquad (5)$$

- **Existence conditions**

From equation (5), the following existence conditions of the sliding mode derive:

$$\begin{cases} a) \quad k^T \cdot (A_2 \overline{x} + b_2 e) \neq 0 \\ b) \quad z_{min} < z_{equiv} < z_{max} \end{cases} \qquad (6)$$

- **Control system with integral controller**

The control system shown in fig.1. has a proportional behavior and, therefore, a stationary error exists. This disadvantage may be removed by adding an I controller, as one can see in fig.2. For the control system with I controller, a new state variable x_R is to de added to the state vector which shall be represented by the same letter $\overline{x}$.

The integral controller is described by the differential equation:

$\dot{x}_R = \left(1/T_i\right) \cdot (w - c^T \overline{x})$, where T_i is the time constant.

The state equation in this case is obtained by adding the state equation of the system without I controller to te differential equation of the integral controller. We get:

$$\dot{\overline{x}} = A\overline{x} + be + b_w w \qquad (7)$$

where:

$$A = \begin{vmatrix} A_s & 0 \\ c_s^T / T_i & 0 \end{vmatrix} ; \ \overline{x} = \begin{vmatrix} \overline{x}_s \\ x_R \end{vmatrix} ; \quad b = \begin{vmatrix} b_s \\ 0 \end{vmatrix} ; \quad b_w = \begin{vmatrix} 0 \\ \frac{1}{T_i} \end{vmatrix} .$$

The index "s" points the variables in the system without the I controller, shown in fig.1.

The switching law, the equivalent control signal and the existence conditions can be obtained in a way similar to the system without I controller.

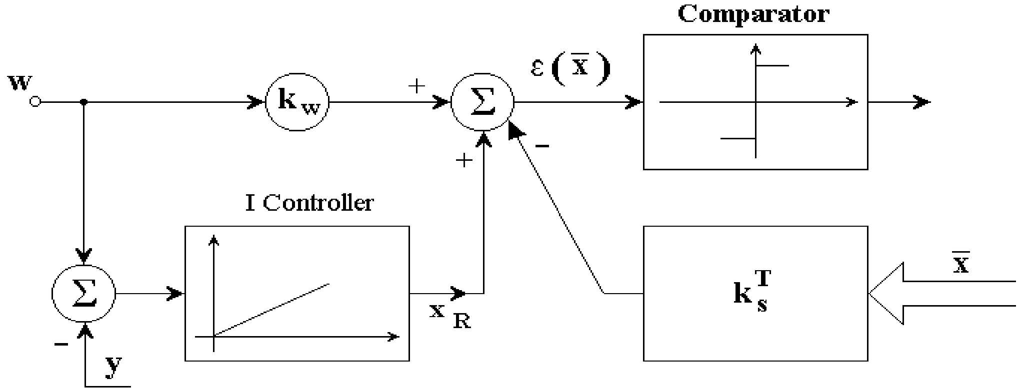

Fig.2. Variable structure control system with I controller

- **The maximum switching frequency**

In fig.3, the comparator hysteresis in terms of the error $\varepsilon(\bar{x})$ (fig.3a) as well as the error $\dot{\varepsilon}(\bar{x})$ variation (fig.3b) are shown. The comparator hysteresis is, in fact, the control signal hysteresis, that is the switching function "z".

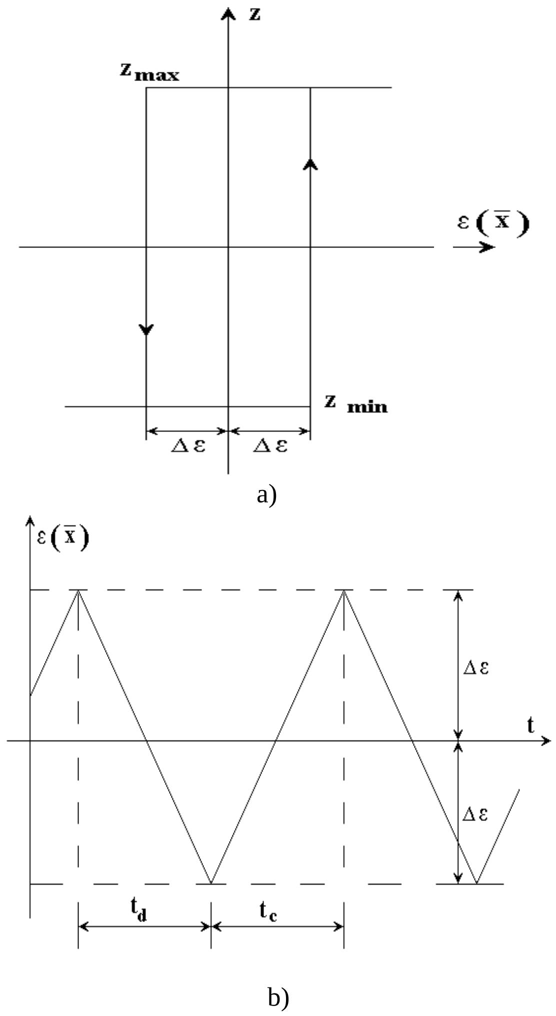

Fig.3. a) the hysteresis of "z"; b) the error variation $\varepsilon(\bar{x})$ (switching law)

In order to get the switching frequency, the control signal (the switching frequency) takes the values z_{min} and z_{max}.

When the reference signal w is constant, its derivative $\dot{w} = 0$. Derivating the switching law given by (4), results:

$$\dot{\varepsilon}(\bar{x}) = k^T \dot{\bar{x}} \tag{8}$$

Taking into account eqs. (2) and (3), a new form for equation (1) results:

$$\dot{\bar{x}} = A_1 \bar{x} + b_1 e + (A_2 \bar{x} + b_2 e) z \tag{9}$$

Substituting eq. (9) in eq. (8) and taking into account that $z = z_{lim}$, we have:

$$\dot{\varepsilon}(\bar{x}) = -k^T [A_1 \bar{x} + b_1 e + (A_2 \bar{x} + b_2 e) z_{lim}] \tag{10}$$

where z_{lim} equals z_{max} or z_{min}.

For $z = z_{equiv}$, $\dot{\varepsilon}(\bar{x}) = 0$, that is:

$$-k^T [A_1 \bar{x} + b_1 e + (A_2 \bar{x} + b_2 e) z_{equiv}] = 0 \tag{11}$$

Subtracting (11) from (10), we get:

$$\dot{\varepsilon}(\bar{x}) = k^T \cdot (A_2 \bar{x} + b_2 e) \cdot (z_{lim} - z_{equiv}) \tag{12}$$

According to fig.3b we may write:

$$t_d = \frac{2\Delta\varepsilon}{-\dot{\varepsilon}(\bar{x})};\ t_c = \frac{2\Delta\varepsilon}{\dot{\varepsilon}(\bar{x})};\ f_c = \frac{1}{t_c + t_d} \tag{13}$$

For determining t_c, z_{lim} is replaced by z_{max} in eq. (12) and for t_d, z_{lim} is replaced by z_{min}.

Using eqs.(12) and (13), the expression of the switching frequency f_c results:

$$f_c = \frac{k^T (A_2 \bar{x} + b_2 e)}{2\Delta\varepsilon} \cdot \frac{(z_{max} - z_{equiv}) \cdot (z_{equiv} - z_{min})}{z_{max} - z_{min}} \tag{14}$$

The switching frequency cancels for $z_{equiv} = z_{max}$ and $z_{equiv} = z_{min}$. It is maximum for:

$z_{equiv} = (z_{max} + z_{min})/2$

and its maximum value is:

$$f_{c_{max}} = \frac{k^T (A_2 \bar{x} + b_2 e)}{8\Delta\varepsilon} \cdot (z_{max} - z_{min}) \tag{15}$$

In the switching frequency expression (15) as well as in the expression of the maximum switching frequency, one can find the term $A_2 \bar{x} + b_2 e$ instead of "b" in the similar formulas given in (Bühler 1986). For $A_2 = 0$ and $b_2 e = b$, the corresponding expressions in (Bühler 1986) are obtained.

The expression "$A_2 \bar{x} + b_2 e$" points out that the switching frequency as well as its maximum value depend on the state vector, that is the maximum switching frequency is not constant.

By means of the block diagram given by Bühler (Bühler 1986), the variable structure nonlinear systems can be analyzed using the sliding mode control; the matrix A is independent of the switching function "z" ($A_2 = 0$) and the input "u" may be written: $u = ze$, that is $b_1 = 0$.

The matrix A is independent of the switching function "z" when the switch and the load S can be separated, that is when there are no reactive elements (inductances or capacitors) between the voltage supply and the switch of the converter, as one can see in the dc-dc Buck converter diagram.

3. APPLICATIONS

3.1. The dc-dc Buck Converter

In fig.4, the block diagram of a dc-dc Buck converter using the sliding mode control for the current i_L is shown.

The switching function is:

$$z = \begin{cases} 1 & \text{if } Tr \text{ is on} \\ 0 & \text{if } Tr \text{ is off} \end{cases}$$

According to the notations in fig.4, for the state variables u_C and i_L we can write the state equation system:

$\dot{\bar{x}} = A\bar{x} + b\,e$,

where:

$$\bar{x} = \begin{vmatrix} u_C \\ i_L \end{vmatrix};\ A = A_1 + zA_2;\ A_1 = \begin{vmatrix} -\dfrac{1}{RC} & \dfrac{1}{C} \\ -\dfrac{1}{L} & 0 \end{vmatrix};\ A_2 = 0$$

$$b = b_1 + zb_2 = z\begin{vmatrix} 0 \\ \dfrac{1}{L} \end{vmatrix};\qquad b_1 = \begin{vmatrix} 0 \end{vmatrix};\qquad e = E$$

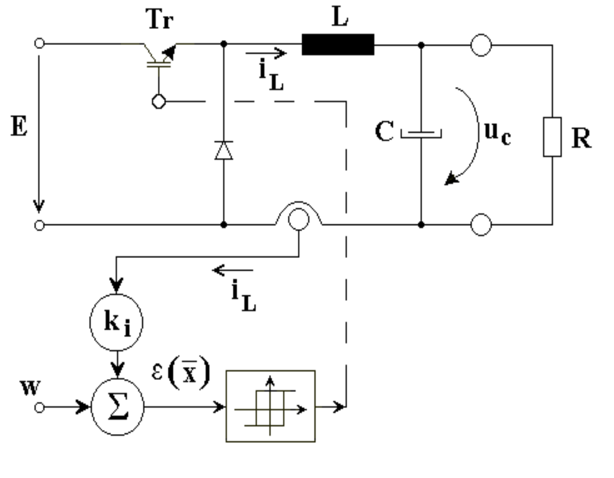

Fig.4. The block diagram of the sliding mode control dc-dc Buck type converter

As $A_2 = b_1 = 0$, the matrix A is independent of the switching function "z" and the input u=ze, hence this diagram may be analyzed in the same way as the block diagram proposed by Bühler (Bühler, 1986).

- **The switching law**

$\varepsilon(\bar{x}) = -k^T\,\bar{x} + w$, where $k^T = \begin{vmatrix} 0 & k_i \end{vmatrix}$

- **The equivalent control signal**

  Using eq. (5), results:

$$z_{equiv} = \frac{u_C}{E}$$

- **The conditions of existence**

  Using eqs. (6), we get:

$$\begin{cases} a)\quad k^T(A_2\,\bar{x} + b_2\,e) = \dfrac{k_i E}{L} \neq 0 \\ b)\quad 0 < \dfrac{u_C}{E} < 1,\quad obviously \end{cases}$$

- **The maximum switching frequency**

  Using eq. (15) and taking into account that $z_{max} = 1$ and $z_{min} = 0$, we get:

$$f_{c_{max}} = \frac{k_i}{L}\cdot\frac{E}{8\Delta\varepsilon}$$

As the converter satisfies the conditions of the block diagram proposed by Bühler, $f_{c\,max}$ is constant and independent of the state variables.

3.2. The dc-dc Boost Type Converter

In fig.5, the block diagram of a dc-dc Boost type converter using the sliding mode control for the control of the current i_L is shown. The circuit may be used, for instance, to design a diode rectifier able to absorb a sine wave current from the electric net, thus improving the power factor; the voltage "e" is the rectified net voltage.

The reference value of the current i^* is the product between a signal proportional to the supply voltage "e" and the signal "k" which is the output of the controller which controls the voltage u_C across the capacitor.

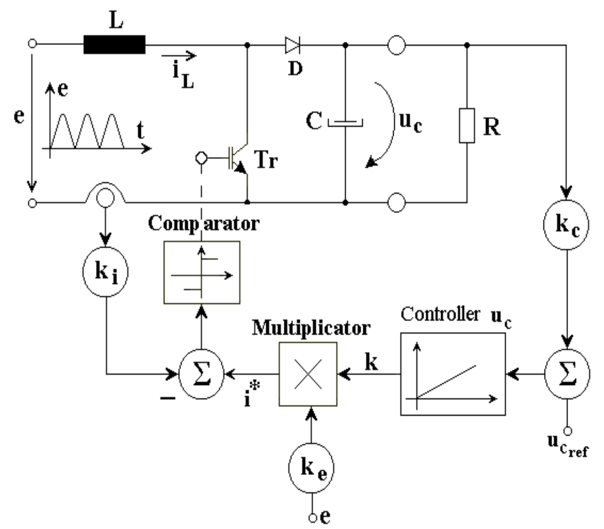

Fig.5. The block diagram of the sliding mode control dc-dc Boost type converter

The switching function is:

$$z = \begin{cases} 0 & if\ \ Tr\ \ is\ on \\ 1 & if\ \ Tr\ \ is\ off \end{cases}$$

According to the notations in fig.5, considering the state variables u_C and i_L, we can write the state equation system:

$\dot{\bar{x}} = A\bar{x} + be$

where:

$$\bar{x} = \begin{vmatrix} u_C \\ i_L \end{vmatrix};\ A = A_1 + zA_2;\ A_1 = \begin{vmatrix} -\dfrac{1}{RC} & 0 \\ 0 & 0 \end{vmatrix};\ A_2 = \begin{vmatrix} 0 & \dfrac{1}{C} \\ -\dfrac{1}{L} & 0 \end{vmatrix};$$

$$b = b_1 + z\,b_2;\qquad b_1 = \begin{vmatrix} 0 \\ \dfrac{1}{L} \end{vmatrix};\qquad b_2 = \begin{vmatrix} 0 \end{vmatrix}$$

and "e" is the rectified sine wave voltage.

Taking into account that the matrix A depends on the switching function "z", as the inductance L separates the supply voltage "e" and the switch Tr, for this diagram we cannot use any more the considerations applicable to the diagram proposed by Bühler.

- **The switching law**

$\varepsilon(\bar{x}) = -k^T\bar{x} + i^*$, where $k^T = \begin{vmatrix} 0 & k_i \end{vmatrix}$

- **The equivalent control signal**

$$z_{equiv} = \frac{1}{u_C} \cdot \left(e - \frac{L}{k_i} \frac{di^*}{dt} \right)$$

- **Existence conditions**

$$\begin{cases} a) \quad k^T \left(A_2 \, \bar{x} + b_2 \, e \right) = \frac{-k_i}{L} u_C \neq 0 \\ b) \quad 0 < z_{equiv} < 1 \end{cases}$$

- **The maximum switching frequency**

Taking into account that $z_{max} = 1$ and $z_{min} = 0$, from (15) results:

$$f_{c_{max}} = \frac{1}{8\Delta\varepsilon} \cdot \frac{k_i}{L} \; u_C$$

Consequently, the switching frequency is proportional to the state variable "u_C".

3.3. Single Phase Inverter Active Filter

In fig.6, the block diagram of a sliding mode control single phase inverter active filter is shown.

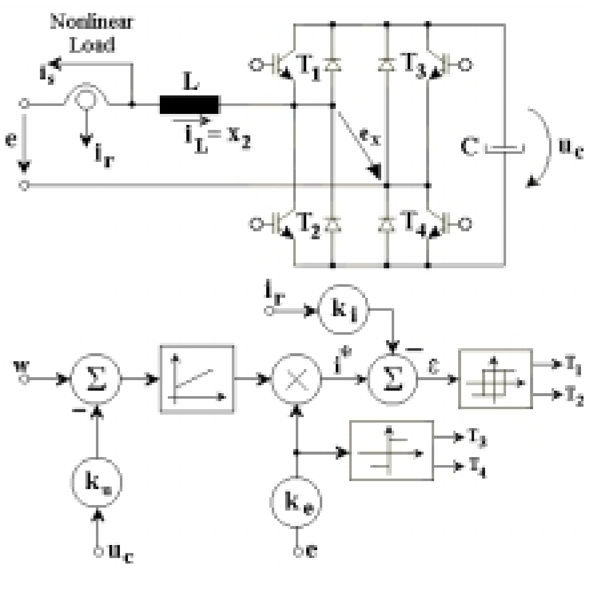

Fig.6. The block diagram of the single phase inverter active filter

- **The switching function**

For the inverter transistors T_1, T_2, T_3, T_4 we define the switching functions z_i (i=1, 2, 3, 4):

$$z_i = \begin{cases} 1 & \text{if } T_i \text{ is on} \\ 0 & \text{if } T_i \text{ is off} \end{cases}$$

- **The operation mode of the inverter**

The operation mode for the T_1, T_2 and for T_3, T_4 transistors is specified by table 1, respectively by tabel 2.

	ε > 0	ε < 0
z_1	0	1
z_2	1	0

Table 1: The operation mode of the transistors T_1 and T_2

	e < 0	e > 0
z_3	1	0
z_4	0	1

Table 2: The operation mode of the transistors T_3 and T_4

From the tables 1 and 2 results that the operation of the transistors T_1, T_2 is determined by the sign of the error $\varepsilon(\bar{x})$ and the operation of the transistors T_3, T_4 is determined by the sign of the supply voltage "e".

When transistors T_1 and T_4 are on, transistors T_2 and T_3 are off and viceversa. Consequently:

$$\begin{cases} z_1 + z_2 = 1 \\ z_3 + z_4 = 1 \end{cases} \tag{16}$$

According to the notations in fig.6, we can write:

$$e_x = \left(z_1 \, z_4 - z_2 \, z_3 \right) u_C \tag{17}$$

Also:

$$z_4 = \frac{1 + sgn(e)}{2} \tag{18}$$

Considering u_C and i_L as state variables and taking into account the eqs. (16), (17) and (18), we can write:

$$\dot{\bar{x}} = A\bar{x} + be$$

where:

$$\bar{x} = \begin{vmatrix} u_C \\ i_L \end{vmatrix}; \quad A = A_1 + z_1 \, A_2; \quad A_1 = \begin{vmatrix} 0 & \frac{sgn(e) - 1}{2C} \\ \frac{1 - sgn(e)}{2L} & 0 \end{vmatrix}$$

$$A_2 = \begin{vmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{vmatrix}; \; b = b_1 + z_1 \, b_2; \; b_1 = \begin{vmatrix} 0 \\ \frac{1}{L} \end{vmatrix}; \quad b_2 = 0$$

e - the sine wave supply voltage

Taking into account that the matrix A depends on the switching function z_1, the conclusions obtained by means of the block diagram given by Bühler are no more valid.

- **The switching law**

According to the notations in fig.6, we get:

$$\varepsilon(\bar{x}) = i^* - k_i i_S - k^T \, \bar{x}$$

where $k^T = \begin{vmatrix} 0 & k_i \end{vmatrix}$

- **The equivalent control signal**

$$z_{equiv} = \frac{L}{u_C} \left(\dot{i}_S - \frac{1}{k_i} \dot{i}^* \right) + \frac{1 - sgn(e)}{2} + \frac{e}{u_C}$$

- **The existence conditions**

$$\begin{cases} a) \quad k^T \left(A_2 \, \bar{x} + b_2 \, e \right) = \frac{-k_i}{L} u_C \neq 0 \\ b) \quad 0 < z_{1_{equiv}} < 1 \end{cases}$$

These conditions are satisfied.

- **The maximum switching frequency**

Takind into account that $z_{1\,max} = 1$ and $z_{1\,min} = 0$, from (15) results:

$$f_{c_{max}} = \frac{1}{8\Delta\varepsilon} \cdot \frac{k_i}{L} u_c$$

Consequently, the switching frequency is proportional to the state variable "u_c".

4. CONCLUSIONS

In the paper a new block diagram for the nonlinear variable structure systems, using the sliding mode control is proposed and discussed; this diagram is much more general than the corresponding one proposed by Bühler (Bühler, 1986).

This block diagram is useful for describing all the nonlinear, variable structure systems which use electronic power converters and can be applied to the nonlinear variable structure systems whose matrices A and b depend, besides the elements (R, L, C) on the switching function "z".

In order to write the state equations of the overall "power converter + load" circuit, we proposed to write everyone of the matrices A and b as a sum of two matrices depending only on the passive elements of the circuit; one of the matrices in the sum is multiplied by the switching function "z".

Starting from the state equatons of the overall "power converter + load" circuit, we obtained:

- the switching law;
- the equivalent control signal;
- the existence conditions for the sliding mode control;
- the maximum switching frequency.

These theoretical results have been applied to the sliding mode control power converters.

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